

Stat 1040 Recitation 6 Solutions

1. (10 points) I buy 24 eggs from a local store. Unknown to me, 6 of the 24 eggs contain salmonella. If I choose 4 eggs at random from the 24 to make a quiche:

(a) What is the chance that none of the 4 eggs contains salmonella?

$$\left(\frac{18}{24}\right) \cdot \left(\frac{17}{23}\right) \cdot \left(\frac{16}{22}\right) \cdot \left(\frac{15}{21}\right) = .288$$

by the multiplication rule

(b) What is the chance that at least one of the 4 eggs contains salmonella?

1 - chance that none contain salmonella  
 = 1 - answer from (a) =  $\left(1 - \left(\frac{18}{24}\right) \cdot \left(\frac{17}{23}\right) \cdot \left(\frac{16}{22}\right) \cdot \left(\frac{15}{21}\right)\right) = .712$

(c) What is the chance that all of the 4 eggs contain salmonella?

$$\left(\frac{6}{24}\right) \cdot \left(\frac{5}{23}\right) \cdot \left(\frac{4}{22}\right) \cdot \left(\frac{3}{21}\right) = .0014$$

(d) What is the chance the second egg I choose contains salmonella?

It's the same as the chance the first egg contains salmonella, so it's  $\frac{6}{24}$ .

2. (5 points) Refer to question 1. Suppose that 25% of all the farms eggs really do contain salmonella. Are we more likely to get more than 30% eggs with salmonella in a sample of 100 or 1000? Explain.

We are looking for a large % error

A large % error is more likely with 100 than 1000.

3. (10 points) A child opens a bag of M&M's and gets 24 candies: 7 red, 3 orange, 5 yellow, 1 brown, 1 green, 2 blue and 5 purple. The child chooses 2 M&M's, at random (without replacement), and gives these 2 M&M's to his sister.

(a) What is the chance that his sister gets two purple M&M's?

$$\frac{5}{24} \cdot \frac{4}{23} = .036$$

(b) What is the chance that his sister gets no purple M&M's?

$$\frac{19}{24} \cdot \frac{18}{23} = .62$$

(c) What is the chance that his sister gets at least one purple M&M?

$$1 - \text{chance of no purples} = 1 - \text{answer to (b)} = .38$$

(d) What is the chance that his sister gets two M&M's that are both purple or two that are both orange?

two purple or two orange (and these are mutually exclusive)

$$\left(\frac{5}{24} \cdot \frac{4}{23}\right) + \left(\frac{3}{24} \cdot \frac{2}{23}\right) = .047$$

4. (10 points) Research shows that there is about a 50% chance that a baby is a girl. Assume that the genders of children in the same family are independent. In a family of four children:

(a) What is the chance that all the children are girls?

$$GGGG \text{ is } \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

(b) What is the chance that not all the children are girls?

$$1 - \text{chance they are all girls} = 1 - \text{answer to (a)} = \frac{15}{16}$$

(c) What is the chance that at least one of the children is a boy?

same as (b)

(d) What is the chance that none of the children are girls?

$$\text{If none are girls, they must all be boys so } \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

(e) What is the chance that all the children are boys?

Same as (d)

(f) What is the chance of getting 2 boys followed by 2 girls?

$$BBGG \quad \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

(g) What is the chance of getting all the same gender?

BBBB or GGGG and these are mutually exclusive

$$\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

5. a)  $\left(\frac{1}{6}\right)^3$

b)  $1 - \left(\frac{1}{6}\right)^3$

c)  $\left(\frac{5}{6}\right)^3$

d)  $1 - \left(\frac{5}{6}\right)^3$

e)

	1	2	3	4	5	6
1	✓					
2		✓				
3			✓			
4				✓		
5					✓	
6						✓

$$\frac{6}{36} = \frac{1}{6}$$

6. (5 points) There are two boxes of tickets - one box has tickets with the numbers 1, 2, 2 written on them. (Yes, there are two tickets with a "2" on). The other box has tickets marked 1,2,3,4. One ticket is drawn at random from each box.

(a) Find the chance that the number from box A is the same as the number from box B.

		1	2	3	4	B		
A	1	✓					$\frac{3}{12}$	
	2		✓					
	2		✓					

(b) Find the chance that the number from box A is greater than the number from box B.

		1	2	3	4	B		
A	1						$\frac{2}{12}$	
	2	✓						
	2	✓						

7. (10 points) An elementary school in Logan employs 15 teachers; 11 are women and 4 are men. Two teachers are selected at random to meet the governor and attend a reception in SLC. Answer each part separately.

(a) What is the probability that both are women?

$$\frac{11}{15} \cdot \frac{10}{14} = \frac{11}{21}$$

(b) What is the probability that at least one is a woman?

$$1 - \text{chance both are men} = 1 - \left( \frac{4}{15} \cdot \frac{3}{14} \right)$$

(c) What is the probability that both are the same gender?

MM or WW and they are mutually exclusive

$$\left( \frac{11}{15} \cdot \frac{10}{14} \right) + \left( \frac{4}{15} \cdot \frac{3}{14} \right) = .581$$

8. (10 points) I have 20 lightbulbs in a large box. Unknown to me, 4 of these 20 bulbs are broken. I select 6 bulbs at random from these 20 bulbs to put in a chandelier. Answer each of the following questions separately.

(a) What is the chance that the first bulb works?

$$\frac{16}{20} = .8$$

(b) What is the chance that the second bulb works?

$$.8 \text{ (just like the first one)}$$

(c) What is the chance that all 6 of the bulbs work?

$$\frac{16}{20} \cdot \frac{15}{19} \cdot \frac{14}{18} \cdot \frac{13}{17} \cdot \frac{12}{16} \cdot \frac{11}{15}$$

9. A pet store has 16 fish in a tank: 7 males and 9 females. I buy 2 of the fish, selected at random.

(a) What is the chance that both of my fish are males?

$$\begin{array}{c} M \quad M \\ \left(\frac{7}{16}\right)\left(\frac{6}{15}\right) \end{array}$$

(b) What is the chance that both of my fish are females?

$$\begin{array}{c} F \quad F \\ \left(\frac{9}{16}\right)\left(\frac{8}{15}\right) \end{array}$$

(c) What is the chance that one of my fish is male and the other is female?

$$MF \text{ or } FM \quad \left(\frac{7}{16}\right)\left(\frac{9}{15}\right) + \left(\frac{9}{16}\right)\left(\frac{7}{15}\right)$$

(d) What is the chance that neither of my fish are male?

same as (b)

(e) What is the chance that at least one of my fish is male?

$$1 - \text{chance neither is male} = 1 - \left(\frac{9}{16}\right)\left(\frac{8}{15}\right)$$